

# Analysis of Auto Parametric Oscillations at the Subharmonic Frequency in Two-Phase Ferro Resonance Circuits

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**Abstract:** Oscillatory processes in nonlinear multiphase electro ferromagnetic circuits (EFMC) have an exceptional variety and complexity, and therefore their research is also associated with solving complex physical problems. Theoretical analysis, in order to identify the main patterns of excitation of subharmonic oscillations of various orders, and the development of engineering calculation methods are of particular importance in designing and creating various switching-type converter devices. On the other hand, two-phase self-oscillating circuits are physical models of power transmission lines (power lines). Consequently, the study of the excitation of the existence of subharmonic oscillations in two-phase systems allows us to establish some patterns of overvoltage with power lines with capacitive compensation caused by harmonic oscillations, and, if possible, take measures to prevent these abnormal modes or mitigate their negative consequences. This article discusses the process of excitation of subharmonic oscillations  $\omega/2$  in two-phase EFMC. Using the method of energy relations, the areas of existence and critical values of the circuit parameters are determined

## 1 INTRODUCTION

It is known that self-oscillations can occur in circuits with electro-ferromagnetic oscillatory circuits. These fluctuations are caused by periodic changes in the nonlinear inductance. Since the change in the nonlinear inductance occurs under the influence of the power supply, the circuit is called auto-parametric [1-7].

In general, auto-parametric oscillations (APO) can be excited in circuits with a nonlinear resistor, inductance, and capacitance due to the ability of circuits to accumulate a certain amount of energy in their elements, which is necessary to excite and maintain the APC at a particular frequency [8-15]. However, the most interesting from a practical point of view are auto-parametric (AP) circuits with an electro ferromagnetic oscillatory circuit capable of developing high power during energy conversion. Ferromagnetic elements in combination with linear capacitances can form circuits in which a significant amount of stored energy during the excitation of the APO can be effectively used to create devices and devices of converter technology.

Of particular interest in this regard are the issues of the development of the theory and methods of calculating auto-parametric (AP) energy converters, which could be the basis for engineering calculations of specific devices.

Of even greater practical interest is the creation of devices based on multiphase AP circuits with an electro ferromagnetic oscillatory circuit (in particular, two-phase and three-phase). Unlike existing energy converters, these devices are made mainly on the basis of single-phase nonlinear circuits, have phase-specific features, and are reliable and easy to operate.

## 2 METHODS AND MATERIALS

Consider the analysis of the two-phase frequency divider in Figure 1, which is two identical ferroresonance circuits consisting of counter-coupled nonlinear inductors of series-connected capacitors. The secondary windings of nonlinear inductors are connected in series and shunted by a

diode to create a constant magnetic field necessary to obtain even harmonics.

When the device is connected to the power source of the generating voltage  $U_1$  and  $U_2$  and at certain ratios between the input voltages and circuit parameters, auto-parametric oscillations are excited in them at the frequency of the second-order subharmonics. At the same time, the phases of excited oscillations are shifted by  $180^\circ$  (Figure 1 b), since the magnetic fluxes created by the currents of the primary and secondary windings of nonlinear inductances are added in one core, while in the other core they are subtracted due to the specified connection of the secondary windings.

Shunting of secondary windings by diode D provides self-magnetization of cores by rectified current.

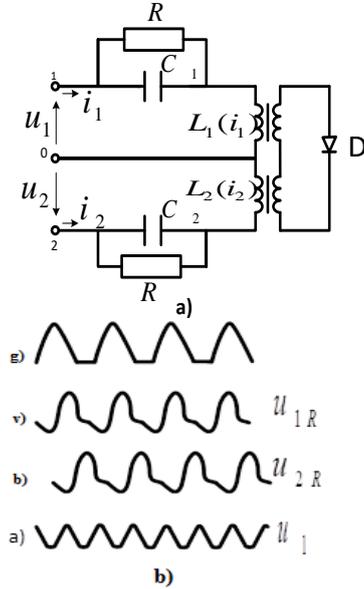


Figure 1: Two-phase frequency divider in two times: a) schematic diagram of the frequency divider; b) voltage waveforms in the tank.

Thus, the device allows for self-excitation of the APO at the second-order subharmonic frequency due to parametric changes in the inductance of a nonlinear reactive element.

### 3 RESULTS AND DISCUSSION

Subharmonic oscillations can be excited in the system, the amplitudes of which are equal but differ in phase by  $180^\circ$  (Figure 1 b) [16].

Depending on this, there are two modes of excitation of the SGC:

$$\Psi_{am} = \Psi_{bm} \quad 0^\circ \text{ and } 180^\circ$$

$$\Psi_{am} = -\Psi_{bm} \quad 180^\circ \text{ and } 0^\circ$$

To analyze the steady-state regime of changes in magnetic fluxes in ferromagnetic elements (FE), we take as:

$$\begin{aligned} \Psi_a &= \Psi_{0a} + \Psi_{1m} \cos(\omega t + \varphi_1) + \Psi_{2m} \cos(\omega t + \varphi_2) \\ \Psi_b &= \Psi_{0b} + \Psi_{1m} \cos(\omega t + \varphi_1 - 180^\circ) + \Psi_{2m} \cos(\omega t + \varphi_2 - 180^\circ) = \Psi_b - \Psi_{1m} \cos(\omega t + \varphi_1) - \Psi_{2m} \cos(\omega t + \varphi_2) \end{aligned} \quad (1)$$

The magnetization characteristic of nonlinear elements is also approximated by an incomplete polynomial of the third degree:

$$i_a = a\Psi_a + b\Psi_a^3, \quad i_b = a\Psi_b + b\Psi_b^3. \quad (2)$$

Substituting (1) into (2) and neglecting terms other than the frequencies  $\omega$  and  $2\omega$ , we obtain expressions for currents (3):

$$i_a = A_{0a} + A_{1a} \cos(\omega t + \varphi_1) + A_{2a} \sin(\omega t + \varphi_1) + B_{1a} \cos(2\omega t + \varphi_2) + B_{2a} \sin(2\omega t + \varphi_2) \quad (3)$$

$$i_b = A_{0b} + A_{1b} \cos(\omega t + \varphi_1) + A_{2b} \sin(\omega t + \varphi_1) + B_{1b} \cos(2\omega t + \varphi_2) + B_{2b} \sin(2\omega t + \varphi_2),$$

where:

$$A_{0a} = a\Psi_0 + b\Psi_0^3 + \frac{3b}{2}\Psi_0\Psi_1^2 + \frac{3b}{2}\Psi_0\Psi_1^2 - \frac{3b}{4}\Psi_1^2\Psi_2 \cos(2\varphi_1 - \varphi_2)$$

$$A_{1a} = a\Psi_1 + \frac{3b}{4}\Psi_1^3 + 3b\Psi_0^2\Psi_1 - \frac{3b}{2}\Psi_1\Psi_2^2 + 3b\Psi_0\Psi_1\Psi_2 \cos(2\varphi_1 - \varphi_2)$$

$$A_{2a} = 3b\Psi_0\Psi_1\Psi_2 \sin(2\varphi_1 - \varphi_2)$$

$$B_{1a} = a\Psi_2 + \frac{3b}{4}\Psi_2^3 + 3b\Psi_0^2\Psi_2 + \frac{3b}{2}\Psi_1^2\Psi_2 + \frac{3b}{2}\Psi_0\Psi_1^2 \cos(2\varphi_1 - \varphi_2)$$

$$B_{2a} = -\left[\frac{3b}{2}\Psi_0\Psi_1^2 \sin(2\varphi_1 - \varphi_2)\right]$$

$$A_{0b} = a\Psi_0 + b\Psi_0^3 + \frac{3b}{2}\Psi_0\Psi_1^2 + \frac{3b}{2}\Psi_0\Psi_2^2 - \frac{3b}{4}\Psi_1^2\Psi_2 \cos(2\varphi_1 - \varphi_2)$$

$$A_{1b} = -\left[a\Psi_1 + \frac{3b}{4}\Psi_1^3 + \frac{3b}{2}\Psi_0^2\Psi_1 + \frac{3b}{2}\Psi_1\Psi_2^2 - 3b\Psi_0\Psi_1\Psi_2 \cos(2\varphi_1 - \varphi_2)\right]$$

$$A_{2b} = 3b\Psi_0\Psi_1\Psi_2 \sin(2\varphi_1 - \varphi_2)$$

$$B_{1b} = -\left[a\Psi_2 + \frac{3b}{4}\Psi_2^3 + 3b\Psi_0^2\Psi_2 + \frac{3b}{2}\Psi_1^2\Psi_2 + \frac{3b}{2}\Psi_0\Psi_1^2 \cos(2\varphi_1 - \varphi_2)\right]$$

$$B_{2b} = -\left[\frac{3b}{2}\Psi_0\Psi_1^2 \sin(2\varphi_1 - \varphi_2)\right].$$

The voltage on nonlinear inductors (4)

$$\begin{aligned} u_a &= \frac{d\psi_a}{dt} = -\omega\Psi_{1m}\sin(\omega t + \varphi_1) - 2\omega\Psi_{2m}\sin(2\omega t + \varphi_2) \\ u_b &= \frac{d\psi_b}{dt} = \omega\Psi_{1m}\sin(\omega t + \varphi_1) + 2\omega\Psi_{2m}\sin(2\omega t + \varphi_2). \end{aligned} \quad (4)$$

Let 's express currents and voltage in a complex form (5):

$$\begin{aligned} \dot{I}_{a1m} &= (jA_{1a} + A_{2a}) e^{j\varphi_1} \\ \dot{I}_{a2} &= (jB_{1a} + B_{2a}) e^{j\varphi_2} \\ \dot{I}_{b1} &= (jA_{1b} + A_{2b}) e^{j\varphi_1} \\ \dot{I}_{b2} &= (jB_{1b} + B_{2b}) e^{j\varphi_2} \\ U_{a1} &= -\omega\Psi_{1m} e^{j\varphi_1} \\ U_{a2} &= -2\omega\Psi_{2m} e^{j\varphi_2} \\ U_{b1} &= \omega\Psi_{1m} e^{j\varphi_1} \\ U_{b2} &= 2\omega\Psi_{2m} e^{j\varphi_2}. \end{aligned} \quad (5)$$

Then the complexes of full capacities are expressed accordingly (6):

$$\begin{aligned} \hat{S}_{a1} &= \frac{1}{2} \dot{U}_{a1} \dot{I}_{a1} \\ \hat{S}_{a2} &= \frac{1}{2} \dot{U}_{a2} \dot{I}_{a2} \\ \hat{S}_{b1} &= \frac{1}{2} \dot{U}_{b1} \dot{I}_{b1} \\ \hat{S}_{b2} &= \frac{1}{2} \dot{U}_{b2} \dot{I}_{b2} \end{aligned} \quad (6)$$

or power for subharmonics (7):

$$\hat{S}_{a1} = \frac{1}{2} (-\omega\Psi_{1m} e^{j\varphi_1}) \cdot (A_{1a} + jA_{2a}) e^{j\varphi_1} \quad (7)$$

$$\hat{S}_{b1} = (\omega\Psi_{1m} e^{j\varphi_1}) \cdot (A_{2b} + jA_{1b}) e^{-j\varphi_1} \quad (8)$$

$$\begin{aligned} P_{a1} &= -\frac{3b}{2} \omega \cdot \Psi_0 \Psi_1^2 \Psi_2 \sin(2\varphi_1 - \varphi_2) \\ Q_{a1} &= \frac{\omega}{2} (a\Psi_1^2 + \frac{3b}{4} \Psi_1^4 + 3b\Psi_0^2 \Psi_1^2 + \frac{3b}{2} \Psi_1^2 \Psi_2^2 + 3b\Psi_0 \Psi_1^2 \Psi_2 \cos(2\varphi_1 - \varphi_2)) \\ P_{b1} &= \frac{1}{2} \omega \Psi_{1m} 3b\Psi_0 \Psi_1^2 \Psi_2 \sin(2\varphi_1 - \varphi_2) \\ Q_{b2} &= -\frac{\omega}{2} [a\Psi_1^2 + \frac{3b}{4} \Psi_1^4 + 3b\Psi_0^2 \Psi_1^2 + \frac{3b}{2} \Psi_1^2 \Psi_2^2 - 3b\Psi_0 \Psi_1^2 \Psi_2 \cos(2\varphi_1 - \varphi_2)] \\ P_{a2} &= \frac{1}{2} \omega \Psi_{1m}^2 \cdot 3b\omega \cdot \Psi_0 \Psi_2 \sin(2\varphi_1 - \varphi_2) \\ P_{b2} &= -\frac{\omega}{2} \Psi_{1m}^2 3b\Psi_0 \Psi_2 \sin(2\varphi_2 - \varphi_1). \end{aligned} \quad (8)$$

According to  $\sum_{k=1}^2 P_k(t) = 0$  for two-phase EFMC, the necessary condition for converting the energy of the frequency  $2\omega$  into the energy of the frequency  $\omega$  will be (9):

$$\begin{aligned} [1, 2, 3] \\ P_{a1} &= -P_{a2} \\ P_{b1} &= -P_{b2} \end{aligned} \quad (9)$$

or

$$\begin{aligned} 0^\circ < 2\varphi_{1a} - \varphi_{2a} < 180^\circ, \\ 180^\circ < 2\varphi_{1b} - \varphi_{2b} < 360^\circ. \end{aligned}$$

By entering the notation (10):

$$\begin{aligned} \alpha &= b\Psi_0\Psi_2\sin(2\varphi_1 - \varphi_2); \\ \beta &= a + \frac{3b}{4} + 3b\Psi_0^2 + 3b\Psi_2^2; \\ \gamma &= 2\varphi_1 - \varphi_2 \\ K_{p1} &= \frac{2P}{\omega\Psi_1^2} = -\alpha \cdot \sin\gamma \\ K_{q1} &= \frac{2Q}{\omega\Psi_1^2} = \beta + \alpha \cdot \cos\gamma \\ K_{p2} &= \alpha \cdot \sin\gamma \\ K_{q2} &= -\beta - \alpha \cdot \cos\gamma. \end{aligned} \quad (10)$$

Squaring and adding them, and using simple pre-formations, we obtain second-order equations with respect to the squares of the amplitudes of the magnetic fluxes  $\Psi_{1m}^2$  and  $\Psi_{2m}^2$ , where  $\Psi_{1m}^2$  is the magnetic flux of the subharmonics ( $\omega/2$ ) and  $\Psi_{2m}^2$  is the component of the magnetic flux of the fundamental harmonic ( $\omega$ ):

$$\begin{aligned} 4\Psi_2^4 + \Psi_1^4 + 16\Psi_0^4 + 4\Psi_2^2\Psi_1^2 + \\ 8\Psi_1^2\Psi_0 - \frac{16}{3b}(K_q - a)\Psi_2^2 - \frac{8}{3b}(K_q - a)\Psi_1^2 - \\ \frac{32}{b}(K_q - a)\Psi_0^2 + \frac{16}{9b^2}[K_p^2 + (K_q - a)^2] = 0, \end{aligned} \quad (11)$$

where  $K_p = \frac{\omega R}{Z^2}$ ;  $K_q = \frac{1}{\omega c Z^2}$ .

Since in the scheme under consideration Figure 1a subharmonic oscillations has phase shifts  $\varphi = 180^\circ$  with symmetric amplitude, then further analysis of (11) is sufficient to study the process of excitation of SGC in one phase.

To determine the quantitative ratios of the subharmonic oscillations mode in (11), the square of the flow of the magnetizing effect, ( $\Psi_0^2$ ) which is the control parameter of the frequency division mode, is considered to be set. In this case, (11) will take the form and will also represent a second-order curve:

$$\begin{aligned} 4\Psi_{2m}^4 + \Psi_{1m}^2 + 4\Psi_{2m}^2\Psi_{1m}^2 - \frac{16}{3b}(k_q - a - 3b\Psi_0^2) \\ \Psi_{2m}^2 - \frac{8}{3b}(k_q - a)\Psi_{2m}^2 + \\ \frac{16}{9b^2}(k_q - a - 3b\Psi_0^2)[(K^*)^2 + 1] = 0 \end{aligned} \quad (12)$$

where:  $K^* = \frac{K_p}{K_q - a - 3b\Psi_0^2}$ .

Invariants that are equal to (13)

$$\begin{aligned} \delta &= \begin{vmatrix} 4 & 2 \\ 2 & 1 \end{vmatrix} S = 4 + 1 = 5 > 0 \\ \Delta &= -\frac{64}{3b} \left[ \left( \frac{K_p}{K_q - a - 3b\Psi_0^2} \right)^2 + 1 \right] \cdot \Psi_0^2 \end{aligned} \quad (13)$$

Since  $\Psi_{2m}^2$  and  $\Psi_{1m}^2$  are the squares of the amplitudes of the main and subharmonic components of the flow couplings, for the existence of the subharmonic oscillations in the system, it is necessary that the curve describing the mode in question (Figure 2) be located on the first quadrant of the plane  $\Psi_{2m}^2 O \Psi_{1m}^2$ . This is possible only under the condition that [19-21] (14).

$$\Psi_0^2 > 0, \quad (14)$$

that is, with a positive magnitude of the magnetizing effect on the ferromagnetic element.

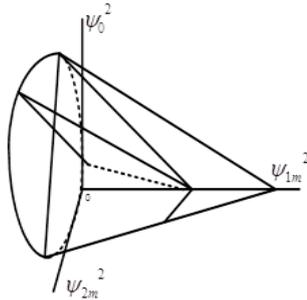


Figure 2: Graphical representation of (12).

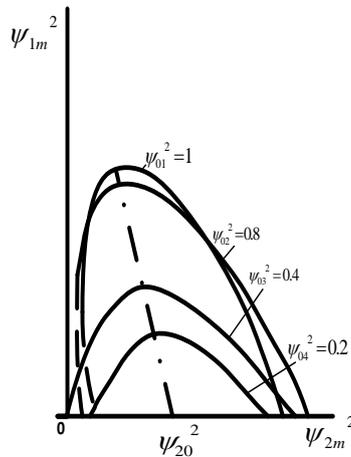


Figure 3: Graphical solution of (12).

If conditions (14) are met, (12) describes real parabolas in the plane  $\Psi_{2m}^2 O \Psi_{1m}^2$ , the coordinates of the vertices of which are determined from the expressions (15)

$$\Psi_{20}^2 = \frac{4}{3b} (K_q - a); \quad \Psi_{10}^2 = \frac{2}{3b} (K_q - a). \quad (15)$$

The angle of rotation of the axes of the parabola relative to the coordinate axes is equal to

$$\alpha = \arctg \left( \frac{-4}{2} \right) \approx -63^\circ 30'.$$

The parameter of the parabola is determined from the expression (16)

$$P = \frac{-24(K_q - a)[-1 - 3b\Psi_0^2]}{3a \cdot b \cdot \sqrt{5}}. \quad (16)$$

From where it can be seen that it is proportional to the magnetizing effect. The construction of the dependence according to (12) is shown in Figure 3.

For the existence of the SGC, it is necessary that the coordinates of the vertices are in the first quadrant of the plane  $\Psi_{2m}^2 O \Psi_{1m}^2$ .

Therefore, the conditions for the existence of SGC are inequalities (17):

$$\Psi_{20}^2 > 0 \quad \Psi_{10}^2 > 0. \quad (17)$$

Since  $\Psi_0^2 > 0$  and  $\Psi_{20}^2 > 0$ , then from (17) the necessary condition for the existence of the SGC will be  $\Psi_{10}^2 > 0$ . Taking into account (15) and (16), (12) acquires the canonical form of writing in the new coordinate system  $(\Psi_{2m}^2)$  and  $(\Psi_{1m}^2)$  (18)

$$(\Psi_{1m}^2)' = P(\Psi_{2m}^2). \quad (18)$$

Since the second-order subharmonic oscillations are excited at positive values of the input effect from (11), it is possible to determine the region of existence by the input effect. Equating  $\Psi_{1m}^2 = 0$  we have (19) and (20):

$$4\Psi_{2m}^4 - M\Psi_{2m}^2 + F = 0, \quad (19)$$

here

$$M = \frac{16}{3b} (K_q - a).$$

$$G = \frac{16}{9} \left( \frac{K_q - a - 3b\Psi_0^2}{b} \right)^2 [(K')^2 + 1], \quad (20)$$

then (21)

$$\Psi_2^2 = \frac{-\sqrt{M^2 - 16F}}{4} \quad (21)$$

or (22)

$$\Delta\Psi_2^2 = -\sqrt{\frac{256}{9b^2} (K_q - a)^2 - \frac{64}{9b^2} (K_q - a - 3b\Psi_0^2)^2 (K')^2}. \quad (22)$$

The critical value of the parameters of the soft excitation circuit of the subharmonic oscillations can be determined from the condition  $\Delta\Psi_2^2 = 0$ , while (23)

$$4(K_q - a)^2 - (K_q - a - 3b\Psi_0^2)^2 (K')^2. \quad (23)$$

It is important to determine in the region of the existence of a second-order subharmonic oscillations bounded by an ellipse according to (12), the region of soft and hard excitation, i.e. determining the limit of variation of the magnetizing effect leading to soft excitation of a soft oscillation.

The condition for soft excitation will be the positivity of the derivative  $a(\Psi_{1m}^2) / a(\Psi_{2m}^2)$  at the intersection point of the left branch of the parabola with the axis of the abscissa. The intersection point

is determined from (12). To do this, we will solve these equations with respect to the input effect

$$(\Psi_{2m}^2) = \frac{16}{3b} (K_q - a) \pm \sqrt{\frac{16}{9b^2} (K_q - a)^2 - 16 \left[ \frac{16}{9b^2} (K_q - a - 3b\Psi_{02}) \right] [(K^1)^2 + 1]} \quad (24)$$

Solving the inequality  $d\Psi_{1m}^2 / d\Psi_{2m}^2 \geq 0$  ( $\Psi_{2m}^2$ )<sub>1</sub> at the point 0 determined from (24), we obtain an expression for the magnetizing effect in the plane  $K_p \theta \Psi_0^2$ , in which a soft excitation mode is possible:

$$4\Psi_0^4 - \frac{32}{3b} (K_q - a) \Psi_0^2 + \frac{64}{9b^2} (K_q - a)^2 \left[ \frac{Kp^2}{(Kq-a)^2} + 1 \right] \geq 0 \quad (25)$$

Expressions (25), when the right side is equal to zero, describes real ellipses in the  $K_p \theta \Psi_0^2$  plane. Coordinates of ellipse centers (26)

$$K_p = 0, \quad \Psi_0^2 = \frac{4}{3b} (K_q - a) \quad (26)$$

Values of the semi-axes (27):

$$aK_q = \frac{16}{3b} (K_q - a), \quad b_{(\Psi_0^2)} = \frac{8}{3b} (K_q - a) \quad (27)$$

To determine the boundary of the soft excitation, we solve (25) with respect to  $\Psi_0^2$ , as a result we obtain (28):

$$(\Psi_0^2)_{12} \geq \frac{8}{3b} (K_q - a) \pm \sqrt{\frac{64}{9b^2} (K_q - a)^2 - \frac{16}{9b^2} (K_q - a)^2 - [(K^1)^2 + 1]} \quad (28)$$

From here it can be seen that the area of “soft” excitation is limited: from below and from above by the values ( $\Psi_0^2$ ). The graph of the separation of the areas of “soft” and “hard” excitation is shown in Figure 4.

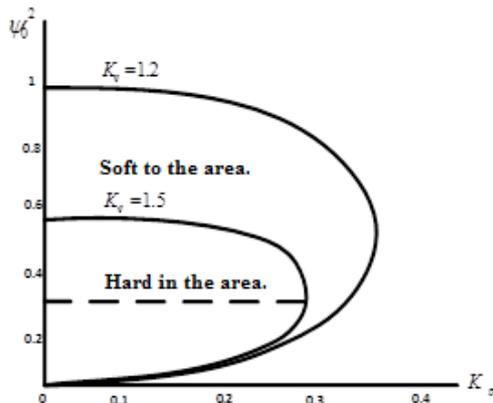


Figure 4: Separation of the areas of “soft” and “hard” excitation is shown.

## 4 CONCLUSION

The paper has accomplished the following:

- 1) Using frequency–energy relations, an equation is obtained that characterizes the steady-state mode of existence of the second-order SGC.
- 2) The results of the analysis of the obtained equations show that the second-order subharmonic oscillations are excited “gently” at certain ratios of the circuit parameters, input voltage and magnetization current.
- 3) The dependence  $Kp = f(Kq)$  is obtained, characterizing the value of the converted power by a reactive nonlinear element to the frequency of the subharmonic oscillations from the power consumption. The analysis of the dependence  $K_p = f(K_q)$  also makes it possible to determine the critical values of the circuit parameters characterizing the region of existence of the subharmonic oscillations of a particular frequency close to the engineering calculation method.

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